Math 1552, Integral Calculus, Summer 2020
Review for Quiz 1

- The first quiz covers the material so far *through* integration by substitution.
- Make sure that you have *Honorlock* setup on your computer well in advance of your Studio section on Thursday!
- The quiz will take place *synchronously* at 11:55AM during the last 20 minutes of studio. Be prepared to attend studio and work problems with your TA before the quiz from 11AM-11:54AM.
- NO notes and NO calculators are allowed on the quiz!
- See examples of the quiz format at the end of this review sheet.
- This review sheet is NOT comprehensive – see your lecture notes and studio worksheets to make sure you study all of the problem types we may ask you about on the quiz.

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1 Review problems

1.1 General comprehension

- Explain why
$$-\int_a^b |f(x)|dx \leq \left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx.$$
- Suppose that $\sum_{k=1}^{n} a_k = 11$ and $\sum_{k=1}^{n} b_k = 6$. Find the value of the sum $\sum_{k=1}^{n} (2a_k + 3b_k)$.
- Which of the following functions are even? Which are odd? *(Why?)*

$$\pm x, x^2, x^3, x^n; \sin x, \cos x, \tan x, \sec x;$$

$$\frac{x^5+2x^3+x}{(x^2+1)^3}, \frac{x^5+2x^3+1}{(x^2+1)^2}, \frac{x^5+2x^3+x}{(1-x)^3}, \frac{x^5+2x^3+x}{(1-x)^6}$$

1.2 Riemann sums and the definite integral

- Give an example of why the inequality $L_f < \int_a^b f(x)dx < U_f$ is *FALSE*.
- Approximate the area $\int_{-1}^{3}(x^3+4)dx$ using
  1. L_f with $n = 4$ rectangles;
  2. U_f using $n = 4$ rectangles;
  3. M_f using $n = 2$ rectangles.
- Evaluate the definite integral
$$\int_2^{12} (x-2)^4 dx,$$
using *Riemann sums*. Justify all relevant steps in your solution. Check your answer using the FTC-based methods we have seen to evaluate the integral.
Hint: Note that for all positive integers $n \geq 1$, we have the formula
$$\sum_{k=1}^{n} k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1).$$

1.3 Antiderivatives and the FTC

- Recall the antiderivative formulas you need to memorize.
- Recall the FTC (both versions and the chain rule variant from lecture).
- Recall the MVT for integrals.
- Find $F'(2)$ if
$$F(x) = \int_{\frac{8}{x}}^{x^2} \left(\frac{t}{1-\sqrt{t}} \right) dt.$$
- Evaluate: $\int \left(\sqrt{x} - \frac{1}{x}\right)^3 dx.$
- Evaluate: $\int \left(\frac{2}{3x} - \frac{1}{\sqrt{16-x^2}} \right) dx.$
- Evaluate: $\int_0^1 \left[8^{-2x} + e^{-3x}\right] dx.$
- Evaluate: $\int_1^2 \frac{3x-5}{x^3} dx.$
- Evaluate: $\int_2^5 (2-x)(x-5) dx.$
- **(Harder – fair game!)** Evaluate the following integral:
$$\int_{-1}^{1} \frac{2x^9 + 3x^5 + 4x^4 + 8x^2 + 4}{(1+x^2)^3} dx.$$
Explain why your solution is correct.

1.4 Integration by substitution

- Evaluate: $\int \frac{\tan\left(x^{1/3}\right)}{x^{2/3}} dx.$
- Evaluate: $\int_{-\frac{\pi}{6}}^{\frac{2\pi}{3}} x^7 \sec^2\left(x^8\right).$
- Evaluate: $\int_0^1 \left(\frac{e^{\sqrt{5}} + x^{\sqrt{5}}}{\sqrt{x}} \right) dx.$

2 Examples of multiple choice type quiz questions

Sample question 1.

Suppose that we are asked to evaluate the following indefinite integral:

$$\int \frac{dx}{1+x^6}.$$

Which of the following answers is correct?

(A) We can evaluate it directly

(B) Evaluate after the u-sub $u = x^3, du = 3x^2dx$

(C) Evaluate after the u-sub $u = x^6, du = 5x^5dx$

(D) We do not know of an elementary solution to this problem (yet)

Sample question 2.

Suppose that we are asked to evaluate the following indefinite integral:

$$I = \int \frac{x^2}{1+x^6}dx.$$

Which of the following is an accurate formula for I?

(A) $\tan^{-1}(x^2) + C$

(B) $\ln(1+x^6) + C$

(C) $\frac{6x^5}{(1+x^6)^2}$

(D) $\frac{1}{3}\tan^{-1}(x^3) + C$

(E) We do not know of an elementary solution to this problem (yet)